Cooperative Spectrum Sensing and Detection for Cognitive Radio

Jayakrishnan Unnikrishnan and Venugopal V. Veeravalli
Dept of Electrical and Computer Engineering
Coordinated Science Laboratory
University of Illinois at Urbana-Champaign

***Abstract*—One of the main requirements of cognitive radio systems is the ability to reliably detect the presence of licensed primary transmissions. Previous works on the problem of detection for cognitive radio have suggested the necessity of user cooperation in order to be able to detect at the really low signal-to-noise ratios experienced in practical situations. We consider a system of cognitive radio users who cooperate with each other in trying to detect licensed transmissions. Assuming that the cooperating nodes use identical energy detectors, we model the received signals as correlated log-normal random variables and study the problem of fusing the decisions made by the individual nodes. We design a linear-quadratic (LQ) fusion strategy based on a deflection criterion for this problem, that takes into account the correlation between the nodes. Our simulation results show that the LQ detector significantly outperforms the counting rule, which is the fusion rule that is obtained by ignoring the correlation.**

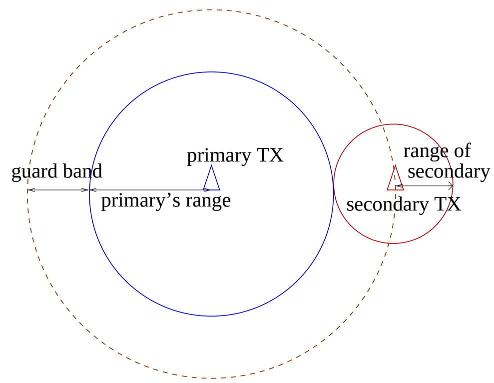

Fig. 1. Guard band. The interior of the primary's range and the guard band together form the protected region.

I. Introduction

Cognitive radio systems that utilize unused *spectral holes* [2] within licensed bands have been proposed as a possible solution to the spectrum crisis. The idea is to detect times when a specific licensed band is unused at a particular place and use the band for transmission without causing any significant interference to the transmissions of the license-holder.

While detecting the presence of a particular transmission is in itself a well-studied communication problem, the specific case of cognitive radio introduces many more constraints on the detection system which makes it a lot more involved. Firstly, the SNR of the signal from the licensed users (referred to as *primary users*) received by the cognitive users (also called *secondary users*) can be extremely small. This is because the secondary users have to ensure that they do not interfere even with the primary transmissions at the edge of the primary's coverage area. As illustrated in Fig. 1, the secondary users located within the primary's range or in the *guard band* around it (jointly referred to as the *protected region*) could potentially interfere with the primary's transmissions. Hence secondary users even at the edge of the guard band should be able to detect the primary signal even if decoding the signal may be impossible [1].

Secondly, the secondary users are, in general, not aware of the transmission scheme used by the primary users and may not be synchronized to the primary's signal. This means that the secondary users are constrained to use non-coherent energy detectors or feature detectors which have much poorer performances than coherent receivers under low SNR. Added to these issues of low SNR is the hidden-terminal problem that arises because of shadowing. Secondary users may be shadowed away from the primary's transmitter but there may be primary receivers close to the secondary users that are not shadowed from the primary transmitter. Hence, if the secondary transmits, it may interfere with the primary receiver's reception. This issue also needs to be addressed in order to design practical solutions to the detection problem.

The issue of low SNR could be addressed by averaging over longer durations of time while performing the detection. This scheme results in an increased effective SNR and hence in improved performance. However, in doing so, we are compromising on another desirable feature of a cognitive radio, which is *agility* or the ability to quickly detect spectral holes. Another possibility as suggested in [1] is to have the primary transmitter send a known pilot signal whenever it is ON. But this may not be feasible since it would require the license holders who own the band to redesign their transmit scheme throughout their network.

An alternative approach is to have users cooperating with each other to detect the primary's signal. Having multiple cooperating users increases diversity by providing multiple measurements of the signal and thus guarantees a better performance at low SNR. Additionally, having users cooperating over a wide area, also provides us with a possible solution to the hidden-terminal problem since secondary users separated

This research was supported in part by the NSF award CCF 0049089, through the University of Illinois.

by a distance larger than the *correlation distance* [5] of the shadow fading are unlikely to be shadowed simultaneously from the primary.

Previous works on user-cooperation for cognitive radio systems ([2], [3]) have studied schemes where some kind of joint detection is employed among all the cooperating users. Gathering the entire received data at one place may be very difficult under practical communication constraints. Moreover, in practice, cooperation between the cognitive radio users cannot be guaranteed in general, since a user can cooperate with others only when there are other users in its vicinity monitoring the same frequency band as itself. We consider a more feasible system like the hard-decision strategy considered in [3] and [4], where the individual secondary users make independent decisions about the presence of the primary in the frequency band that they are monitoring and communicate their decisions to a fusion center. The fusion center makes the final decision about the occupancy of the band by fusing the decisions made by all cooperating radios in that area that were monitoring the same frequency band. In practice, the fusion center could be some centralized controller that manages the channel assignment and scheduling for the secondary users. The system could also be one where the secondary users exchange their decisions and each secondary user performs its own fusion of all the decisions. We assume that each secondary user knows the geographic locations of the other users and hence the correlation between the observations. They are however unaware of the primary's location. Since the decisions made by the secondary users contain just one bit of information each, and since we do not expect to have to keep track of the channel usage frequently, the data rates required for reliably communicating these observations are expected to be within practical limits. Furthermore, the duration of data transmission is also not expected to affect the *agility* of the spectrum sensing system.

In this paper, we address the problem of fusing the decisions made at the cooperating sensors. We note that for the cognitive radio application we would have to deal with the fact that the sensors are going to observe conditionally dependent data due to correlated shadowing. The main contribution of this paper is a suboptimal fusion rule that handles correlation issues and at the same time is not heavily dependent on the model or on exact knowledge of the statistics of the signal. The rule that uses only the knowledge of lower order moments of the quantized data is shown to give good performance in correlated environments. It has the added advantage that the design of the individual users' detectors can be done by taking into account only that users' own signal statistics. The individual sensors can be oblivious of the number of sensors or the correlation between the sensors. These details need be known only at the fusion center where the final decision is taken. In the subsequent sections we first introduce the problem formulation followed by our proposed solution and then present our results and conclusions.

II. Problem Formulation

The basic task of the fusion center is to decide whether or not the secondary users are located inside the protected region shown in Fig. 1. As mentioned earlier, we assume that the secondary users employ energy detectors. Since the cooperating secondary users are expected to be located close to each other and are monitoring the same frequency band, the distributions of the received powers they see can be modeled as being identical, albeit not independent. So the problem now becomes a binary hypothesis testing problem to decide whether or not the mean received power at their location is higher than the power expected at the outer edge of the protected region. We model the received power as being log-normally distributed when the primary is ON and the secondary users are within the protected region. The log-normal distribution is a popular choice for modeling shadowing in wireless systems [14] in which it is assumed that the received power in dB is distributed as Gaussian. We also adopt the popular correlation model [5] in which the correlation between the powers in dB received at two different sensors decays exponentially with the distance between them.

When the secondary users are outside the protected region or when the primary is switched OFF, the power they receive would be dominated by the noise. Under this scenario, the output of the energy detectors will be proportional to the noise power or variance. However, perfect knowledge of the noise powers is not feasible in practice due to the uncertain interfering signals in the environment. We therefore model this uncertainty by modeling the received signal as being log-normal distributed with some known variance as in [15]. Furthermore, we assume that the uncertainties are *i.i.d.* (independent and identically distributed) across the sensors.

The cooperative detection system of the secondary users should guarantee that it successfully detects the presence of primary transmissions as long as the users are within the protected region. But it is sufficient to ensure that the system does not cause any interference for the worst scenario which would occur when the secondary receivers are located at the edge of the guard band. In essence, we are adopting a *robust detection* approach. Hence, we now have the two hypotheses for the distribution of the primary SNRs relative to the noise floor at the cooperating nodes in dB:

$$\begin{aligned} H_0 : \underline{Y} &\sim \mathcal{N}(\mu_0 \underline{1}, \Sigma) \\ H_1 : \underline{Y} &\sim \mathcal{N}(\underline{0}, \sigma_1^2 I) \end{aligned} \tag{1}$$

where μ_0 is the mean net power (signal plus noise) received at the edge of the guard band when the primary is present expressed in dB relative to the noise floor, σ_1 quantifies the uncertainty in noise power, $\underline{1}$ is the vector of all ones, Σ is the matrix with elements $\Sigma_{ij} = \sigma_0^2 \rho^{d_{ij}}$ where d_{ij} is the distance between nodes indexed by i and j, ρ is a measure of the correlation-coefficient between nodes separated by unit distance, and σ_0^2 is the net variance under H_0. The parameter ρ is related to the correlation distance $\mathsf{D_c}$ [5] by the relation $\rho = \exp(-\frac{1}{\mathsf{D_c}})$. Any modeling inaccuracies can be compensated by assigning a smaller value to μ_0. Here H_0 is the hypothesis that the primary is present and the secondary users are located within the protected region, and H_1 is the hypothesis that the primary is absent or is far away from the secondary users. We use p_0 for the distributions under H_0 and p_1 for the distributions under H_1. The constraint that our system should meet is to guarantee that the probability of interfering with the primary transmissions is less than some specified limit α. Thus this problem is essentially the Neyman-Pearson binary hypothesis testing problem [6] where we aim to maximize the probability of detection under H_1 subject to a constraint on the allowed false-alarm rate under H_0 because we further assume that the secondary users could transmit whenever they see that the spectrum is unoccupied. In this paper, the probability of detection refers to the probability of detecting free spectrum and the probability of false alarm refers to the probability of incorrectly declaring that the spectrum is free when it is actually occupied.

The distributions described in (1) above are those of the received powers at the secondary nodes in dB. In practice, however, the fusion center has access only to decisions $\{U_i\}_1^n$ made by the individual sensors based on their individual observations $\{Y_i\}_1^n$. In the presence of other cooperating nodes this bit U_i is communicated to the fusion center and the fusion center takes the final decision $\delta(\underline{U})$ about the hypothesis based on the bits received from all the sensors.

III. Detection rule at the nodes

Since each cognitive radio is not designed expecting cooperation from other users in the detection process, the detector employed at each user would meet the false alarm constraint individually. Moreover, since the distributions of the signals received at each sensor are assumed to be identical, the detectors they use will also be identical. The optimal test used by the ith sensor to determine its decision U_i will be a likelihood ratio test on its observation Y_i of the form:

$$U_i = I_{\{\log(L(Y_i))>\tau\}}$$

where $I_{\{\}}$ is the indicator function and $L(Y_i) = \frac{p_1(Y_i)}{p_0(Y_i)}$ is the likelihood ratio of the observation at the ith node. The threshold τ is chosen so as to meet the false alarm rate with equality at each node separately. Setting U_i to 1 is considered equivalent to deciding in favor of H_1. For the Gaussian hypotheses described in (1), the log-likelihood ratio of the observations will be a quadratic function of Y_i in general [6]. Hence the ith node would have to compare a quadratic function of its observation to a threshold and obtain its decision in the form of bit U_i and communicate it to the fusion center.

IV. Fusion of decisions

Fusion of data observed at distributed locations is an integral part of any decentralized detection procedure. However most of the significant works on decentralized detection have focused on the cases with conditionally *i.i.d* observations (see e.g. [8] and [9] for an overview of these results). The correlated case has also been studied [10] but the results are often not very easy to implement in practice.

The optimal rule for fusing observations obtained from distributed sensors [10] [16] is to compute the joint likelihood ratio of the bits and compare it with a threshold chosen so as to meet the false alarm rate requirement. This solution, in general, requires the knowledge of the joint statistics of the quantized observations under both hypotheses. However, in our problem the U_i's are quantized versions of correlated Gaussian variables under H_0 and hence their joint statistics are not easy to compute especially for large values of n. In this paper, we present some simple suboptimal fusion strategies that provide a solution to this problem. The structure of these detectors can be obtained using only partial statistical information about the quantized observations. However, setting the threshold at the fusion center corresponding to a target false alarm rate will need to be done using simulations. This would be the case with all possible detectors since analytical expressions for the error probabilities of all detection rules can be obtained only when the joint statistics of the observations are available.

A. Counting Rule

One of the simplest suboptimal solutions to the data fusion problem is the *Counting Rule* [11] (also referred to as the *Voting Rule*), which just counts the number of sensor nodes that vote in favor of H_1 and compare it with a threshold. The threshold value has to be set using simulations since the joint statistics under H_0 are not available. It is easy to see that under the special scenario where the observations are $i.i.d$ across the sensors under both hypotheses, this is the optimal rule since the joint likelihood ratio of the bits in that case would be a monotonic function of the number of nodes voting in favor of H_1. This would hence be a reasonable thing to do even when nothing is known about the correlation structure. How well a rule designed for a decentralized hypothesis test with correlated observations makes use of the correlation information could thus be quantified by comparing its performance with that of the counting rule for the same observations.

B. Linear Quadratic detector

In this section we present the main contribution of this paper - a general suboptimal solution to the fusion problem that uses partial statistical knowledge and gives better performance than the one obtained by ignoring the correlation information completely. This solution makes use of up to the second order statistics of the local decisions $\{U_i\}_1^n$ under H_0 and up to the fourth order statistics under H_1, in the form of moments. Since the observations are independent under H_1, the moments under H_1 are easily calculated or estimated. The second order moments under H_0 can be obtained by calculating or estimating just the pairwise statistics under H_0. We note that obtaining information about these moments is in general a lot easier than obtaining the entire joint statistics of the signals especially when there are a large number of cooperating nodes.

We look at detectors in the class of linear-quadratic (LQ) detectors, i.e. detectors that compare a linear-quadratic function of $\{U_i\}_1^n$ with a threshold to make a decision about the hypothesis. Since we are including quadratic terms as well while computing our detection metric, we expect to see improved performance over the Counting Rule that was purely linear. Moreover, since we are using only moment information about $\{U_i\}_1^n$, this detector is quite general and can be used for all classes of distributions of the signals. We seek to optimize over the class of LQ detectors using the *generalized signal-to-noise ratio* or *deflection* criterion [13]. The deflection of a detection rule that compares any function $T(\underline{X})$ of the observations $\underline{X}$ with a threshold is defined as:

$$D_T = \frac{[E_1(T(\underline{X})) - E_0(T(\underline{X}))]^2}{\text{Var}_1(T(\underline{X}))} \tag{2}$$

where H_1 is the noise-only hypothesis in our case. A detector with a higher value of deflection is expected to have better error-probability performance than one with a lower value of deflection. Although deflection cannot be related directly to the error probability for non-Gaussian observations, we use simulations to demonstrate the error-performance of the optimal deflection-based LQ detector that we derive in this section.

Following [12], we solve for the optimal LQ detector. Since we now have quadratic terms as well, the values that we assign to the bits become significant. The decisions $\{U_i\}_1^n$ contribute to the hypothesis test only through their probabilities under the two hypotheses. This observation suggests that an intelligent choice of values to be assigned to the decision variables in our problem would be the log-likelihood ratios of the bits themselves.

Hence we express our decision metric as:

$$T(\underline{X}) = \underline{h}^T \underline{X} + \underline{X}^T M \underline{X} \tag{3}$$

where $\underline{X}$ is the vector of log-likelihood ratios of the received bits with means under H_1 subtracted, given by

$$X_i = \log\left(\frac{p_1(U_i)}{p_0(U_i)}\right) - E_1\left[\log\left(\frac{p_1(U_i)}{p_0(U_i)}\right)\right]$$

while $\underline{h}$ is a vector of length n and M is an n-by-n square matrix. We need to find the optimal LQ metric of the form (3) that maximizes the deflection given by (2). Clearly, this optimization will require the knowledge of up to the second order statistics of the bits under H_0 and up to the fourth order statistics of the bits under H_1 since these terms explicitly appear in the expression for the deflection (2).

Define matrix $C = E_1[\underline{X}\,\underline{X}^T]$. Since adding a constant to the decision metric leaves the deflection unchanged, (3) can be replaced by a new decision metric given by:

$$S(\underline{Z}) = \underline{p}^T \underline{Z} \tag{4}$$

where $\underline{p}$ is now an (n^2+n)-by-1 vector and $\underline{Z}$ is an (n^2+n)-by- 1 vector given by:

$$\begin{aligned} \underline{Z} = [\ X_1 \ldots \quad X_n \qquad & X_1^2 - C_{11} \ldots \quad X_1X_n - C_{1n} \\ & X_2X_1 - C_{21} \ldots \quad X_2X_n - C_{2n} \\ & \qquad \ldots \\ & X_nX_1 - C_{n1} \ldots \quad X_n^2 - C_{nn}\ ]^T \end{aligned}$$

In other words, we form $\underline{Z}$ by appending $\underline{X}$ with the raster-scanned form of $\underline{X}\,\underline{X}^T - C$. So the first n elements of $\underline{Z}$ are the elements of $\underline{X}$, the next n are the elements of the first row of $\underline{X}\,\underline{X}^T - C$, followed by the elements of the second row of $\underline{X}\,\underline{X}^T - C$ and so on. Similarly, $\underline{p}$ can be viewed as a vector formed by appending vector $\underline{h}$ with matrix M in raster-scanned form. So the problem of finding optimal $\underline{h}$ and M reduces to solving for the optimal $\underline{p}$ that maximizes the deflection for this decision metric.

From the construction of $\underline{Z}$ it is easy to see that $\underline{Z}$ has zero mean under H_1. Hence, applying (2) to (4), we have deflection for $S(\underline{Z})$ given by:

$$D_S = \frac{(\underline{p}^T \underline{\mu})^2}{\underline{p}^T K \underline{p}} \tag{5}$$

where $\underline{\mu} = E_0(\underline{Z})$ and $K = E_1(\underline{Z}\,\underline{Z}^T)$. In general, matrix K is positive semi-definite and not strictly positive definite. But it can be shown that the vector $\underline{p}$ that drives the denominator of (5) to zero drives the numerator also to zero. This means that the two distributions of $S(\underline{Z})$ have the same mean. Since we do not desire this, we optimize (5) only over those $\underline{p}$ vectors that do not lie in the singular space of matrix K. This optimization can be performed in a straightforward manner similar to the approach in [12] as long as matrix K and vector $\underline{\mu}$ are known. We do not include the details here due to lack of space. Since the elements of matrix K involve only the second, third and fourth order moments of X_i's under H_1, this optimization can be performed as long as these lower order statistics of the decision variables can be calculated.

V. Simulation Results

Since analytical expressions for the error probabilities of these detectors cannot be obtained, we need to resort to simulations for estimating their performances. The performance of a detector can be illustrated using its *receiver operating characteristic* (ROC) [6] which is the plot of the detection probability under H_1 against the false alarm probability under H_0 obtained with the detector. If the ROC of one detector lies above that of another at a particular value of false alarm probability, it means that the former detector performs better than the latter for a Neyman-Pearson test at that level. For our detection problem, the detection probability under H_1 is given by $p_1(\delta(\underline{U}) = H_1)$ and the probability of false alarm under H_0 is given by $p_0(\delta(\underline{U}) = H_1)$ where δ represents the final decision about the hypothesis taken at the fusion center.

A. Uniform node location in a unit square

The two rules obtained in the previous section were simulated for a network of 9 cooperating nodes uniformly placed inside a unit square with the distance between nearest neighbors kept at one-half. The correlation parameter ρ appearing in the Σ matrix in (1) was kept at $\rho = 0.6$. This basically amounts to assuming that the length of the side of the square is around half the correlation distance since $\exp(-0.5) \simeq 0.6$. Assuming a mean received SNR of -3dB at the edge of the guard band, a shadowing standard deviation of 4dB and a noise uncertainty, σ_0, of 1dB, we get the value of the mean total power at the edge of the guard band, μ_0, to be 2.2 dB and the effective standard deviation of the received power under H_1, σ_1, to be 1.6dB.

We simulated the two rules for false-alarm probability values in the range 0.01 to 0.04. The nodes use identical likelihood ratio tests for obtaining their decision variables

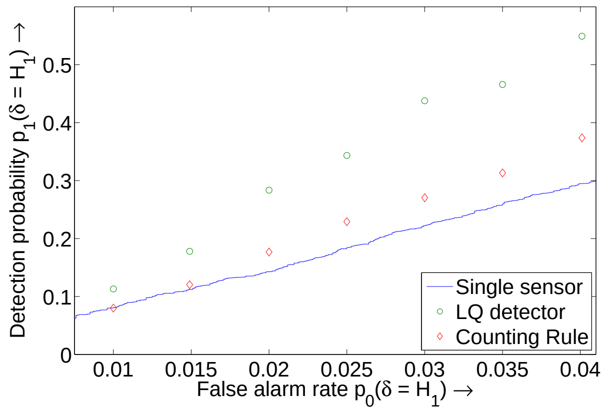

Fig. 2. Comparison of single sensor ROC and detection probabilities obtained with LQ detector and counting rule for 9 sensors uniformly distributed inside a unit square with parameters set to $\rho = 0.6$, $\mu_0 = 2.2$ dB, $\sigma_1 = 1.6$ dB and $\sigma_0 = 1$ dB

TABLE I
TABLE OF ERROR PROBABILITIES

α (false alarm rate)	Detection probability p_d for			
	Single sensor	Counting rule	LQ detector	Centralized rule
0.01	0.080	0.079	0.113	0.618
0.02	0.142	0.177	0.284	0.746
0.03	0.222	0.271	0.438	0.818
0.04	0.295	0.374	0.549	0.871

U_i. The threshold used at the nodes is chosen to achieve equality for the individual false alarm probability. We allow for randomization at the fusion center while performing the detection. The ROC of the single sensor detector is illustrated in (Fig. 2). The points obtained by applying the LQ and counting rule detectors for fusing the decision variables of all sensors are also shown on the same graph. As expected, the performances of the detectors that make use of the information from all the sensors are better than the one that uses decisions made at a single sensor. In particular, the LQ detector is seen to give around twice the detection probability as that of the single sensor detector for the false alarm values considered even though the observations are highly correlated - the distance spanned by the nodes is equal to half the correlation distance of the shadow fading. It can also be inferred that the LQ detector give a substantial gain of around 50% to 60% over the counting rule detector, thus demonstrating the efficiency of the LQ detector. The exact values obtained are also listed in table I. From the table it can be also be seen that the centralized rule that uses the unquantized observations $\underline{Y}$ performs much better than the other detectors that use only the decision variables $\underline{U}$, thus illustrating the loss in performance incurred due to making hard decisions at the cooperating nodes. However, implementing such a centralized detector may not be feasible in practice. It is expected that for a system comprising of a larger number of cooperating nodes, better detection probabilities could be obtained with the LQ detector.

VI. Conclusions

Our results clearly show that it is important not to ignore the correlation between the nodes for fusing the local decisions. The LQ detector provides a simple detector which gives significant performance gains over the counting rule while still using only partial statistical knowledge of the correlated decision variables, thus giving a practical suboptimal solution to the decision fusion problem in a cooperative cognitive radio network. It can also be shown that the LQ detector achieves the performance of the optimal fusion rule for the special case where there are only two cooperating nodes, thus further justifying the efficiency of this detector. We are unable to include this result here due to lack of space, but we plan to include it in the conference presentation.

Acknowledgment

The authors would like to thank Jason Chang for helping out with the simulations.

References

[1] A. Sahai, N. Hoven and R. Tandra, "Some Fundamental Limits on Cognitive Radio", *Allerton Conference on Communication, Control, and Computing*, Oct. 2004.

[2] S. M. Mishra, A. Sahai and R. W. Brodersen, "Cooperative Sensing among Cognitive Radios," in *IEEE International Conference on Communications*, ICC 2006, vol. 4, pp. 1658-1663, June 2006.

[3] E. Visotsky, S. Kuffner and R. Peterson, "On Collaborative Detection of TV Transmissions in Support of Dynamic Spectrum Sharing," in *First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks*, DySPAN 2005, pp. 338-345, Nov. 2005.

[4] A. Ghasemi and E. S. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks*, DySPAN 2005, pp. 131-136, Nov. 2005.

[5] M. Gudmundson, "Correlation model for shadow fading in mobile radio systems," *Electronic Letters*, vol. 27, no. 23, pp. 2145-2146, Nov. 1991.

[6] H. V. Poor, *An Introduction to Signal Detection and Estimation*, 2nd ed. Springer-Verlag, New York, 1994.

[7] J. N. Tsitsiklis, "Decentralized detection," *Adv. Statist. Signal Process.*, vol. 2, pp. 297-344, 1993.

[8] J. F. Chamberland and V. V. Veeravalli, "Decentralized detection in sensor networks," *IEEE Trans. Signal Processing*, vol. 51, no. 2, pp. 407-416, 2003.

[9] V. V. Veeravalli and J.-F. Chamberland, "Detection in sensor networks," to appear in *Wireless Sensor Networks. Signal Processing and Communi. Perspectives*, A. Swami et al (Eds.), Wiley, 2006

[10] E. Drakopoulos and C.-C. Lee, "Optimum multisensor fusion of correlated local decisions," *IEEE Trans. Aerospace and Electronic Systems*, vol. 27, no. 4, pp. 5-14, July 1991.

[11] V. Aalo and R. Viswanathan, "Asymptotic performance of a distributed detection system incorrelated Gaussian noise" *IEEE Trans. Signal Processing*, vol. 40, pp. 211-213, 1992.

[12] B. Picinbono and P. Duvaut, "Optimal linear-quadratic systems for detection and estimation," *IEEE Trans. on Inform. Theory*, vol. 34, no. 2, pp. 304-311, 1988.

[13] B. Picinbono, "On deflection as a performance criterion in detection," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 31, no. 3, 1995

[14] T. S. Rappaport, *Wireless Communications: Principles and Practice*, Prentice Hall, 2002.

[15] K.V. Cai, V. Phan and R.J. O'Connor, "Energy detector performance in a noise fluctuating channel," in *Military Communications Conference*, Milcom 1989, vol. 1, pp. 85-89, Oct. 1989.

[16] M. Kam, Q. Zhu, and W. S. Gray, "Optimal data fusion of correlated local decisions in multiplesensor detection systems," in *IEEE Trans. on Aerospace and Electronic Systems*, vol. 28, no. 3, pp. 916-920, 1992